Reduced-order Decomposition and Coordination approach for Markov-based Stochastic UC with Distributed Wind Farms and BESS

Niranjan Raghunathan, Zongjie Wang†, Bing Yan, Tianqiao Zhao, Meng Yue

***Abstract*—To achieve carbon neutrality, US states are enhancing renewable energy use and encouraging battery integration. This paper addresses the challenges posed by renewable energy uncertainties in resource operation. We formulate unit commitment (UC) with distributed wind generation and grid-scale batteries as a Markov-based stochastic problem. Due to scalability issues with increasing wind farms, we decompose the model into approximate area subproblems using reduced-order models and Principal Component Analysis (PCA). These subproblems are efficiently resolved and coordinated through a Surrogate Absolute-Value Lagrangian Relaxation-based framework. The simulation results on the IEEE 118-bus system with 75% wind penetration level have demonstrated the effectiveness and efficiency of the proposed method at managing these complexities and highlight the potential benefits of integrating batteries.**

***Index Terms*—Distributed wind, Markov processes, Renewable integration, SAVLR, Stochastic UC.**

I. Introduction

In pursuit of decarbonizing the electric grid, New England aims to increase its renewable energy share to 50% by 2040, as per state policies [1]. Wind energy represents 66% of new proposals in the ISO New England Interconnection Queue, followed by solar at 16% and battery energy storage systems (BESSs) at 14%. The stochastic nature of these renewables, however, poses significant challenges to the grid's reliable and cost-efficient operation, especially with such a high proportion of renewables in the resource mix.

In the day-ahead UC process, it is crucial to develop solutions that are not only cost-efficient but also resilient to the uncertainties of renewable generation. Incorporating grid-scale BESSs alongside renewables in the UC stage enhances the process by utilizing their storage capacity and rapid ramping abilities. This integration is particularly effective in improving UC solutions under scenarios of high wind penetration. It is critical to develop accurate modeling and integration of renewable generation uncertainties into the UC formulation, in conjunction with BESSs.

Following this, exploring the array of approaches detailed in literature for managing these uncertainties becomes a key step. The risk-averse approaches, such as Robust Optimization (RO) [2], Interval Optimization (IO) [3], and Chance-Constrained Optimization (CCO) [4, 5], prioritize solution feasibility over cost-effectiveness. RO specifically seeks an optimal solution that is feasible for the worst-case realizations. Although these methods ensure feasibility, they might not always be cost-effective or accurately represent actual variations, and identifying the worst-case scenarios can be challenging. In contrast, risk-neutral approaches like Stochastic Programming (SP) [6] and Markov-based Stochastic UC (MSUC) [7, 8] aim to minimize overall expected costs, making them preferable for achieving cost-efficient solutions. This distinction is critical in selecting an approach that effectively balances cost-efficiency with the ability to handle the uncertainties associated with renewable energy integration. The authors favor MSUC over SP for its more efficient and comprehensive handling of stochasticity [6]. MSUC employs a Markov process that models renewable generation's stochasticity based on current states, independent of past states. This method models stochasticity through states and their transitions over time (termed the Markovian uncertainty set), scaling linearly with the optimization horizon. However, despite its streamlined approach, the number of states in MSUC increases exponentially with the addition of distributed renewables.

While recent literature has advanced in managing uncertainties in renewable energy integration, challenges persist, particularly in UC with distributed wind farms and BESSs. These challenges include the exponential complexity of accurately modeling multiple distributed wind sources and the handling of soft transmission capacity (TC) constraints. To address these issues, this paper proposes a novel reduced-

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order decomposition and composition approach for MSUC, significantly advancing the field. Key innovations and contributions of this paper include:

- Development of a novel reduced-order decomposition and composition approach for MSUC with distributed wind farms, BESSs, and soft TC constraints;
- Creation of area-perspective stochastic models that incorporate local and approximated nonlocal components, using Principal Component Analysis (PCA) for efficient dimensionality reduction;
- Innovative decomposition of the complex MSUC problem into approximate area subproblems (AASPs), effectively reducing the computational complexity;
- Introduction of a novel decomposition and coordination (D&C) framework based on Surrogate Absolute-Value Lagrangian Relaxation [9] (SAVLR-AASP), enhancing solution robustness and cost-effectiveness;
- Demonstrating the effectiveness of the approach through Monte Carlo simulations.

The paper is structured as follows. Section II details the development of the Markovian uncertainty set and MSUC formulation. Section III focuses on decomposing MSUC into AASPs, optimizing local resources through a customized, reduced-order Markov model. Section IV demonstrates a test case using the IEEE 118-bus system with 10 wind farms and 5 BESSs, demonstrating the effectiveness and efficiency of the proposed SAVLR-AASP method. The results highlight its ability to provide robust, cost-effective solutions quickly, emphasizing the significant role of BESSs in reducing wind curtailment, alleviating network congestion, and aiding peak demand management.

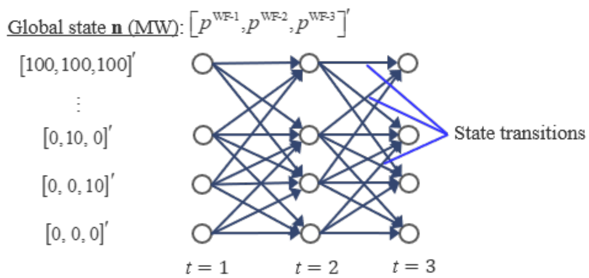

Fig. 1. Markov model for a system with 3 distributed wind farms.

II. MARKOV-BASED STOCHASTIC UC

In Subsection II-A, the paper establishes the Markovian uncertainty set, while Subsection II-B is dedicated to formulating the MSUC.

A. Markovian uncertainty set

The Markovian uncertainty set includes global wind generation states, transitions between states at subsequent time intervals, and their probabilities. These global states represent combinations of discrete wind generation levels across all wind farms. An example of a Markov model for a system with three distributed wind farms is illustrated in Fig. 1. The set of global wind generation states, $\{\mathbf{n}\}$ (where $\mathbf{n}$ is an index for tuples of wind generation levels at each distributed wind farm), the state probabilities, $\varphi_{\mathbf{n}}[t]$, and transition probabilities are obtained by modeling the evolution of wind generation at distributed wind farms as a time-invariant Markov process.

B. Markov-based Stochastic UC

In MSUC, the aim is to identify a set of commitment decisions for conventional generators that minimizes the total expected cost, considering unit-level constraints, system demand, and soft TC constraints. This total cost comprises commitment costs, expected dispatch costs (calculated over all probabilistic global states), and the average soft constraint penalties. Soft TC constraints are critical in this stochastic context, as rigidly meeting hard constraints for every possible wind generation state can be challenging. The approach of using the average of soft constraint penalties, as opposed to expected values, ensures equal weighting across all states, thereby avoiding disproportionately small penalties for constraint exceedances in low probability states. The objective function is formulated based on (9) of [7] and (1) of [10] as:

$$\begin{aligned}
&\sum_{t\in T}\Bigg(\sum_{j\in J}\left(C_j^{\text{SU}}u_j[t]+C_j^{\text{NL}}x_j[t]\right)\\
&+\sum_{\mathbf{n}\in N_{\text{ss}}}\varphi_{\mathbf{n}}\Bigg[\sum_{j\in J}\Bigg(\sum_{o\in O_j}C_{j,o}^{\text{gen}}p_{\mathbf{n},j,o}^{\text{gen}}[t]\Bigg)+\sum_{w\in W}\left(C^{\text{cur}}p_{\mathbf{n},w}^{\text{cur}}[t]\right)\\
&+\sum_{b\in B}\left(C_b^{\text{BESS+}}p_{\mathbf{n},b}^{\text{BESS+}}[t]+C_b^{\text{BESS-}}p_{\mathbf{n},b}^{\text{BESS-}}[t]\right)\Bigg]\\
&+\sum_{\mathbf{n}\in N_{\text{ss}}}\Bigg[\frac{1}{|N_{\text{ss}}|}\sum_{l\in L}C_l^{\text{pen,TC}}\left(\upsilon_{\mathbf{n},l}^{\text{TC+}}[t]+\upsilon_{\mathbf{n},l}^{\text{TC-}}[t]\right)\Bigg]\Bigg),
\end{aligned} \tag{1}$$

where $t, t\in T$ is the time index; unit $j, j\in J$, has the following bounded decision variables: $\{x_j[t]\}$ are binary on/off status variables; $\{u_j[t]\}$ are binary startup variables; $\{p_{\mathbf{n},j,o}^{\text{gen}}[t]\}$ are continuous generation-levels of block o, $o\in O_j$ for state $\mathbf{n}$. $\{p_{\mathbf{n},w}^{\text{cur}}[t]\}$ are continuous wind curtailment levels at wind farm w; $\{p_{\mathbf{n},b}^{\text{BESS+}}[t]\}$ and $\{p_{\mathbf{n},b}^{\text{BESS-}}[t]\}$ are continuous charge and discharge levels of battery b, $b\in B$, respectively; $\{\upsilon_l^{TC+/-}[t]\}$ are continuous variables for soft transmission capacity violations in the positive and negative directions, respectively, for line l, $l\in L$. All the decision variables are nonnegative. Costs and penalties within (1) include startup costs $\{C_j^{\text{SU}}u_j[t]\}$, no-load costs $\{C_j^{\text{NL}}x_j[t]\}$, generation costs $\{C_{j,o}p_{\mathbf{n},j,o}[t]\}$, curtailment costs $\{C^{\text{cur}}p_{\mathbf{n},w}^{\text{cur}}[t]\}$, charge and discharge costs for BESSs $\{C_b^{\text{BESS+}}p_{\mathbf{n},b}^{\text{BESS+}}[t]\}$ and $\{C_b^{\text{BESS-}}p_{\mathbf{n},b}^{\text{BESS-}}[t]\}$, respectively, and soft transmission capacity penalties $\{C_l^{\text{pen,TC}}(\upsilon_{\mathbf{n},l}^{\text{TC+}}[t]+\ \upsilon_{\mathbf{n},l}^{\text{TC-}}[t])\}$.

The problem is subject to the following constraints: Startup and generation limits are formulated as given in (3), (4), and (5-6) from [7], respectively; minimum up/down time constraints are formulated as given in (14) of [11]; ramp up/down constraints model the ramping capability of units within the time interval (e.g., 30 minutes or 1 hour), and couples variables in subsequent time intervals. Due to this coupling, ramping constraints must also consider all possible state transitions between subsequent time intervals. The ramp up/down constraints are respectively formulated as given in (9) and (10) of [7]. For example, ramp up constraints are formulated as:

$$p_{\mathbf{n},j}^{\text{gen}}[t]-p_{\mathbf{m},j}^{\text{gen}}[t-1]\le R_j\cdot x_j[t]+(\underline{P}_j+\frac{R_j}{2})\cdot\left(x_j[t]-x_j[t-1]\right)\ \forall j,\ \mathbf{m},\ \mathbf{n}\in\left\{\mathbf{n}\in\vec{N}_{\mathbf{m}}\mid\pi'_{\mathbf{mn}}>0\right\},\ t \quad (2)$$

where is $p_{\mathbf{n},j}^{\text{gen}}[t]$ total generation of unit j and $\underline{P}_j$ is minimum generation level of unit j when it is online. It also includes constraints for BESSs, such as charging and discharging constraints, SoC tracking, and initial and terminal SoC conditions to ensure adequate charging at the start of each day. System demand constraints are integrated to match the total output from generators, BESSs, and wind farms with the net demand (total demand minus battery charging and wind curtailment) across all global states, in line with (2) from [10]. Lastly, soft TC constraints for all wind generation states are formulated following (3) from [10].

III. Solution Methodology

Subsection IV-A details the development of area-perspective Markov models using a PCA-based method, while Subsection IV-B introduces the novel SAVLR-based decomposition and coordination framework.

A. Area-perspective Markov models

In area-perspective models, the variation of local wind generation is emphasized, while that of nonlocal wind generation is approximated by using PCA to reduce the dimensionality of the nonlocal wind generation components. For each area a, eigendecomposition of the covariance of nonlocal wind generation components is performed to obtain $\mathbf{V}$, the orthogonal matrix of eigenvectors $\mathbf{\Xi}$, the diagonal matrix of corresponding eigenvalues To reduce dimensionality while retaining the maximum overall variance, only the eigenvectors corresponding to the κ largest eigenvalues are retained in $\mathbf{V}$ to obtain $\tilde{\mathbf{V}}$, where κ is selected to balance the tradeoff between computational performance and model accuracy. The eigenvalues and the orientation of eigenvectors depend on the underlying spatio-temporal correlation structure of distributed windfarms. $\tilde{\mathbf{V}}$ spans a κ-dimensional space D, which is a subspace of the space spanned by $\mathbf{V}$. Let D' be an affine subspace of D:

$$D'=\{\mathbf{d}+\boldsymbol{\mu}_{\bar{a}}\mid\mathbf{d}\in D\}, \quad (3)$$

where $\boldsymbol{\mu}_{\bar{a}}$ is the mean of nonlocal wind generation components. Following dimension reduction, only those nonlocal wind generation states that intersect D' are kept for the area-perspective models. An illustration of this dimension reduction process for a system with three wind farms is depicted in Fig. 2.

Dimension reduction might not fully capture the entire range of possible nonlocal wind generation in area-perspective models, so states in $\hat{N}_a^{\text{NL}}$ are linearly scaled for completeness. The global wind generation states for the area a-perspective model, N_a^{AP}, are then formed by combining local states in N_a^{L} and nonlocal states in $\tilde{N}_a^{\text{NL}}$. However, even after dimension reduction, the number of states could be large enough to impact the computational efficiency of AASPs. To address this, a state filtering method is employed, sampling states evenly across quartiles of aggregated generation levels to further streamline the number of states.

B. SAVLR-based D&C framework

To decompose the MSUC, initially, the system demand constraints are relaxed. This is achieved by incorporating the average value of these relaxed constraints with corresponding Lagrangian multipliers, into the objective function, thus forming the surrogate Lagrangian. The use of average value, as opposed to expected values over probabilistic global states, ensures equal weighting of constraint violations across all global states in the Lagrangian, mirroring the approach used for soft penalty terms in (1). Weighing these by state probabilities would disproportionately emphasize penalty costs for high probability states, potentially leading to excessive violations in low probability states in the final solution. Furthermore, to expedite the convergence of multipliers during the iterative solution process, the average of the absolute value of relaxed demand constraints, multiplied by the SAVLR penalty coefficient c, is added to the Lagrangian. The absolute-value terms are linearized in a standard manner, the resulting Lagrangian, inclusive of the SAVLR penalty terms is denoted as L_c. The objective for each AASP a at a given iteration k is then formulated based on this modified Lagrangian framework as:

$$\min_{\{u,x,p^a,\upsilon^a,z^a\}} L^a_{c^k,\lambda^k},\ \text{where}$$

$$\begin{aligned}
L^a_{c^k,\lambda^k} &= \sum_{t\in T}\Bigg(\sum_{j\in J_a}\left(C_j^{\text{SU}}u_{j,k}[t]+C_j^{\text{NL}}x_{j,k}[t]\right)\\
&+\sum_{j\notin J_a}\left(C_j^{\text{SU}}\bar{u}_{j,k-1}[t]+C_j^{\text{NL}}\bar{x}_{j,k-1}[t]\right)\\
&+\sum_{\mathbf{n}\in\tilde{N}_a^{\text{AP}}}\Bigg(\varphi_{\mathbf{n}}\Bigg[\sum_{j\in J}\Bigg(\sum_{o\in O_j}C_{j,o}^{\text{gen}}p_{\mathbf{n},j,o,k}^{\text{gen}}[t]\Bigg)+\sum_{w\in W}C^{\text{cur}}p_{\mathbf{n},w,k}^{\text{cur}}[t]\\
&+\sum_{b\in B}\left(C_b^{\text{BESS+}}p_{\mathbf{n},b,k}^{\text{BESS+}}[t]+C_b^{\text{BESS-}}p_{\mathbf{n},b,k}^{\text{BESS-}}[t]\right)\Bigg]\Bigg)\\
&+\sum_{\mathbf{n}\in\tilde{N}_a^{\text{AP}}}\Bigg(\frac{1}{\left|\tilde{N}_a^{\text{AP}}\right|}\Bigg[\sum_{l\in L}C_l^{\text{pen,TC}}\left(\upsilon_{\mathbf{n},l,k}^{\text{TC+}}[t]+\upsilon_{\mathbf{n},l,k}^{\text{TC-}}[t]\right)\\
&+\lambda_{\mathbf{n},k}^{\text{D}}[t]\left(z_{\mathbf{n},k}^{\text{D,+}}[t]-z_{\mathbf{n},k}^{\text{D,-}}[t]\right)+c_k\left(z_{\mathbf{n},k}^{\text{D,+}}[t]+z_{\mathbf{n},k}^{\text{D,-}}[t]\right)\Bigg]\Bigg)\Bigg),
\end{aligned} \quad (4)$$

where J_a is the set of units in area a, $\{\bar{u}_{j,k-1}\}$ and $\{\bar{x}_{j,k-1}\}$ are the fixed startup and no-load values of nonlocal units, and $\{\lambda_{\mathbf{n},k}^{\text{D},a}\}$ are multipliers of relaxed demand constraints for subproblem *a,* and $\{z_{\mathbf{n},k}^{\mathrm{D},+/-}\}$ are nonnegative linearization variables for linearizing the absolute-value of the demand constraints. AASPs comply with all original problem constraints, yet the extensive number of ramp rate constraints, arising from numerous state transitions, slows down computational performance. To enhance efficiency, only extreme transitions are factored into ramp rate constraints. This approach is justified under the assumption that online units typically adjust their generation levels (ramp down/up) in response to the overall wind generation's fluctuation (ramping up/down). This strategic simplification significantly streamlines the number of transitions considered, thus improving computational speed.

The AASPs are coordinated within an SAVLR-based coordination framework. First, the surrogate optimality condition requirement (S.O.C) (See [12] for more details) is calculated by approximating the dispatch behavior of nonlocal units based on latest available values of their commitment variables. Then, the AASP is solved until satisfaction of the S.O.C, after which, the multipliers belonging to the AASP are updated, thus facilitating coordination with other AASPs. The stopping condition for the algorithm is based on a threshold for the RMS of constraint violations. A flowchart of the algorithm is provided in Fig. 3.

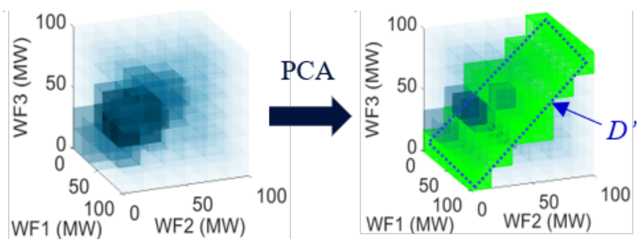

Fig. 2. Example of PCA based dimension reduction.

IV. Simulation Studies

A test case is studied using a modified IEEE 118-bus system, featuring ten wind farms (accounting for 75% wind penetration) and five BESSs with a total capacity of 1025 MW/MWh, distributed across six areas. The efficacy of the SAVLR-AASP method, using 25 global states per AASP (referred to as SA-25), is evaluated against deterministic models for expected and zero wind scenarios (labeled DET-exp and DET-zero, respectively) and a scenario-based SP with 100 scenarios (SP-100).

Solutions are verified through Monte Carlo simulations, utilizing wind generation scenarios derived from the original Markov model, enhanced with importance sampling [13, 14] for increased simulation efficiency. For these scenarios, an economic dispatch problem is solved, based on commitment decisions from the SAVLR-AASP method. In this context, demand constraints are treated as soft, with a high penalty coefficient (e.g., \$10,000/MW), ensuring the feasibility of solutions. This approach guarantees that violations of these constraints occur only in cases where the problem is infeasible otherwise, maintaining solution integrity.

Monte Carlo simulations also test scenario problems both with and without BESSs, assessing their potential to enhance cost-efficiency and operational robustness under high wind penetration conditions. The algorithm's stopping criterion is set at a root mean square (RMS) of demand constraint violations below 300 MW for all AASPs. Testing is conducted on an Intel Xeon CPU 2.6 GHz, 8 Cores, 64 GB laptop, utilizing MATLAB R2018a and CPLEX 12.8. Notably, the original MSUC could not be resolved using the branch-and-cut (B&C) algorithm in CPLEX with default settings, owing to the overwhelming number of variables and constraints required to manage all potential states.

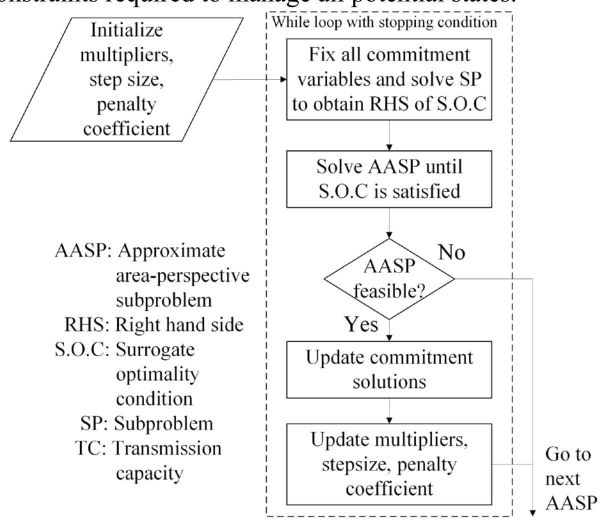

Fig. 3. Flowchart of SAVLR with AASPs.

Table 1 shows that SA-25 delivers a low-cost solution without any constraint violations in under 30 mins. While SP-100 achieves a slightly lower cost solution, it incurs violations in 7 scenarios (with up to 25 MW of transmission capacity and 50 MW of demand violations) and requires over an hour. Thus, SA-25 emerges as the best overall solution, achieved within 27 mins. Scenarios are also solved using the SA-25 commitment solutions but without including BESSs, providing a basis for comparison. Although the actual benefits of BESSs are contingent on their real-time operational strategies, their behavior in these MC simulations suggests significant advantages.

Omitting BESSs leads to approximately 2,000 MW of extra wind curtailment, TC constraint violations in five scenarios, and an average cost increase of \$8,790 per solution. To understand how BESSs contribute to minimizing wind curtailment and enhancing both the robustness and cost-efficiency of operations, an in-depth analysis of their dispatch behavior in Scenario 135 is conducted. Figure 4 illustrates the hourly aggregated operation of BESSs and wind curtailment. For Scenario 135, both with and without BESSs. In this scenario, BESSs effectively absorb the majority of wind energy that would otherwise be curtailed, except for 61 MW in hour 5. This stored energy is later released, contributing to reduced overall operational costs. Notably, the operational

TABLE I: RESULTS FOR 118-BUS SYSTEM WITH 10 WIND FARMS (75% WIND PENETRATION) AND 5 BESSS.

Formulation	Avg. scenario cost	Avg. penalty cost	# of online units (all time)	# of scenarios with constraint violations	Total solution time	MIP gap (CPLEX)	Avg. AASP solve time
DET-zero	\$459,340	\$4,210	142	37	0.52s	0.17%	n/a
DET-exp	\$12,238,129	\$11,839,669	334	873	0.55s	0.14%	n/a
SP-100	\$415,940	\$4,540	215	7	1h 17m	0.26%	n/a
SA-25	**\$417,650**	**\$0**	**245**	**0**	26m 32s	n/a	**13.8s**

cost without BESSs is \$20,000 higher (excluding soft penalties). Further analysis across more scenarios reveals that BESSs play a crucial role in alleviating transmission network congestion. They engage in energy arbitrage by charging during periods of high wind generation and discharging during times of low wind and peak demand, optimizing system efficiency and cost.

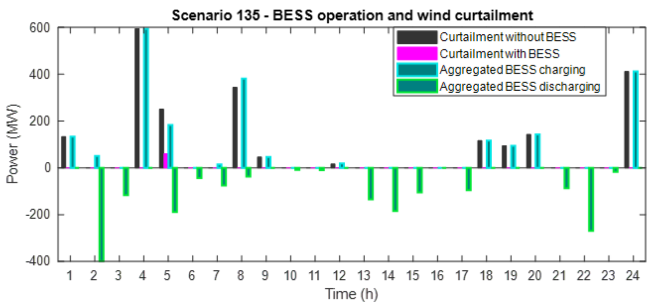

Fig. 4. BESS operation and wind curtailment for Scenario 135.

V. CONCLUSIONS

This paper introduces an innovative decomposition and coordination (D&C) framework for Markov-based stochastic unit commitment (MSUC) with distributed wind and battery energy storage systems (BESS). To manage the complex Markov model for distributed wind generation, this paper first develops a principal component analysis (PCA)-based method that constructs area-perspective models. These models effectively reduce the dimensionality of the original Markov model, tailored to each area's specific context while retaining maximal information. The MSUC is then segmented into approximate area subproblems (AASPs) using these area-perspective models. These AASPs are then iteratively solved, with their solutions being coordinated through the surrogate absolute-value Lagrangian relaxation-AASP (SAVLR-AASP) method. Testing on a modified IEEE 118-bus system with ten distributed wind farms have demonstrated that the SAVLR-AASP approach efficiently achieves low-cost and robust solutions within a 30-minute window, a performance not matched by deterministic models or stochastic programming (SP) with 100 scenarios.

The methodology is scalable for larger systems incorporating ten or more distributed renewable resources, by effectively decomposing the MSUC into manageable subproblems. Moreover, the analysis of BESS operations reveals their significant role in reducing wind curtailment and relieving transmission network congestion. BESSs enhance operational cost-efficiency by charging during periods of high wind and discharging as needed, such as during peak load or low wind periods, especially under high wind generation penetration. This conclusion underlines the efficacy and scalability of our proposed approach in integrating renewables into power systems.

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